**Path Integral Formulation of GF’s**

Might want to refer to the Propagator file for most of this stuff. But let’s say we have the following Hamiltonian,



And we wanted to consider evaluating things like this:



where φH(x,t) is the Heisenberg developed operator, and |GS> the ground state of the interacting problem. The first thing we need to do is to put the vacuum |GS> in terms of an eigenstate of the field operator , i.e., |φ(x)>. This can be done via the mass-gap theorem. Consider the time development of |φa>, and <φb|. We have variously,



where |n> are the eigenstates of the interacting problem. Now in the large-time limit, if the energy levels En are separated, only the lowest energy level term should matter, as the others would wash out in comparison (since their frequency of oscillation is much greater). So we can approximately say:



(taking ta,b → ∞ limit implicitly) Solving for |GS>, we have:



And then have:



Now we can get rid of |GS> by recognizing, from above, that:



so,



and inserting the U’s implicit within (x,t), we have:



where TC orders the operators from 0 to t to t´ to 0. We can now remove the TC operator [need to be more clear on this point], and write:



Now insert resolutions of identity:



(see Propagator file to see how this comes about, or check out QM/Time-Dependent/Propagator or GF path integral files, etc.) and the denominator may be written as a path integral too, giving us,



Now we take the ta,b → ∞ limit explicitly. And when we do, it seems the boundary values disappear. Perhaps that is because in this limit, the starting/ending points have negligible contribution. So we have:



and the d4X integration encompasses all space-time. The Feynman rules derived from this path integral just replicate the ones already discussed. And we’ll note the denominator will cancel the vacuum bubbles in the numerator. But we may still have disconnected diagrams where φ(x,t) and φ(x´,t´) are not connected to each other.